

Quiz 7 Summary

#2) Objective is to find C such that

$$C \int_0^2 \int_0^1 \frac{x(1+x)}{b} dx dy = 1.$$

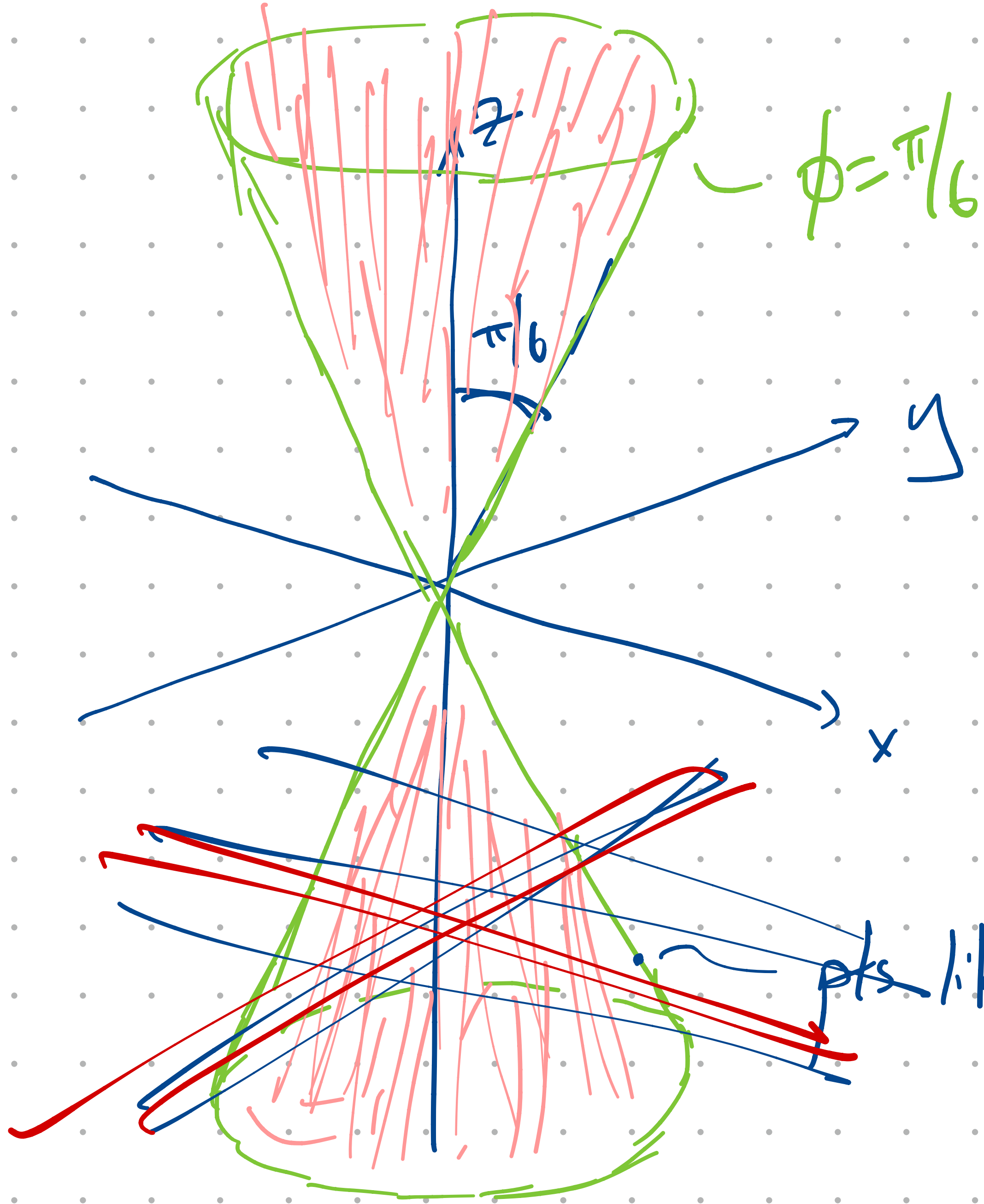
#1)

$$\int_0^{2\pi} \int_0^{\pi/6} \int_0^{\sec \phi} \rho^2 \sin \phi d\rho d\phi d\theta$$

$0 \leq \theta \leq 2\pi$ (this covers everything)

$$0 \leq \phi \leq \pi/6$$

$$0 \leq \rho \leq \sec \phi$$



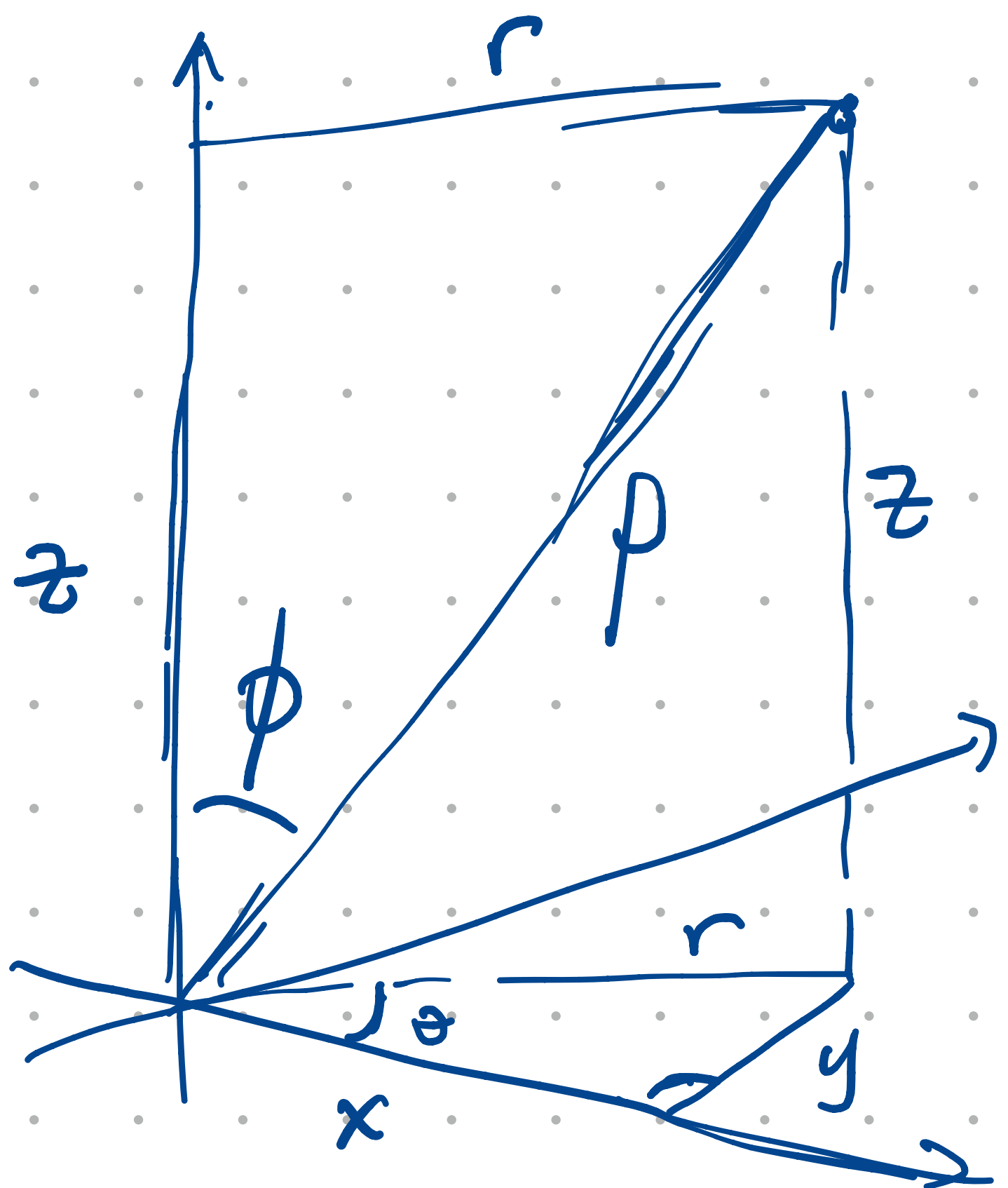
We have $\rho \geq 0$

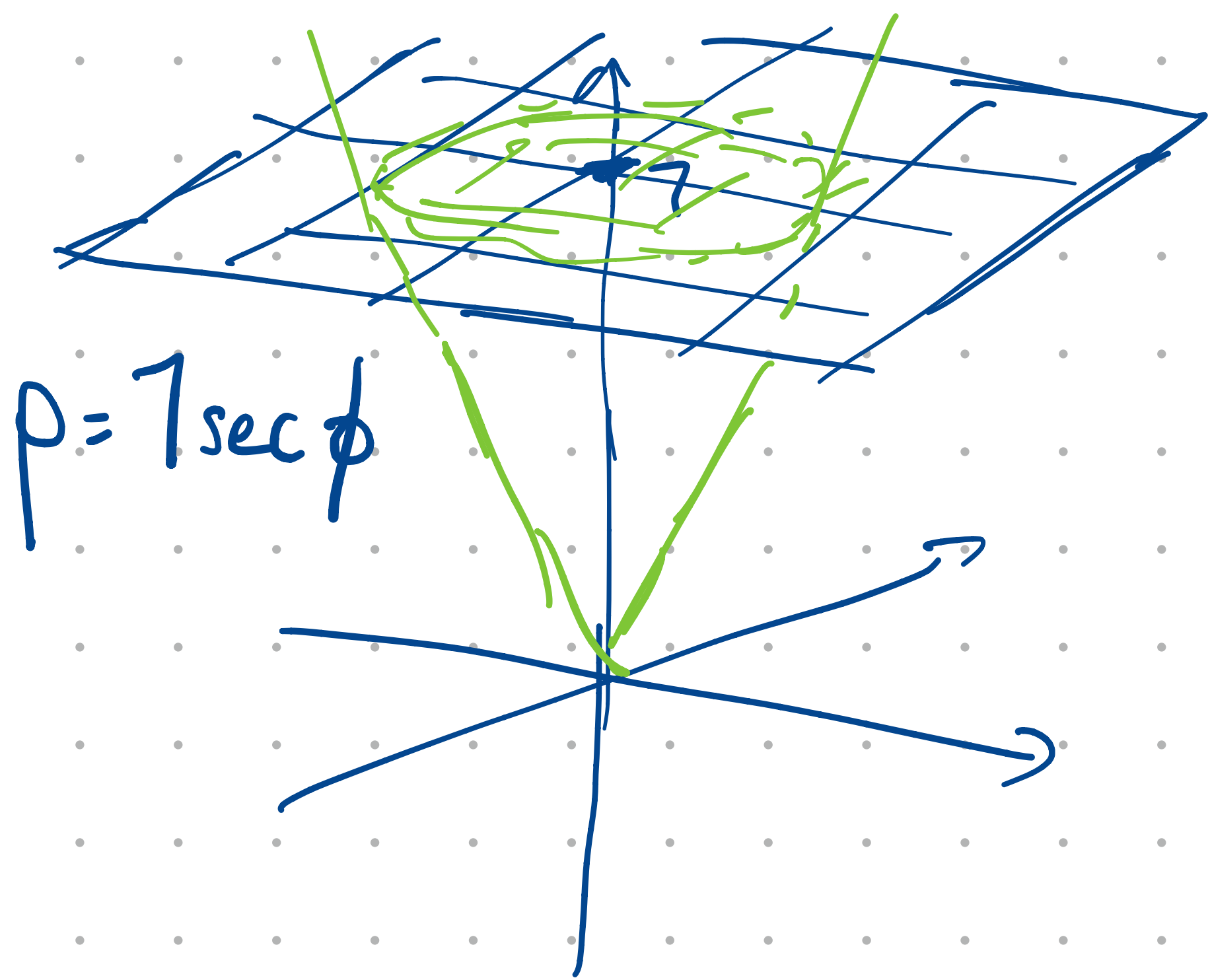
pts like this can be represented with $\phi = \pi/6$
 $\rho < 0$
 or $\phi = 5\pi/6$ and $\rho > 0$.

$$\rho \cos \phi \leq 7$$

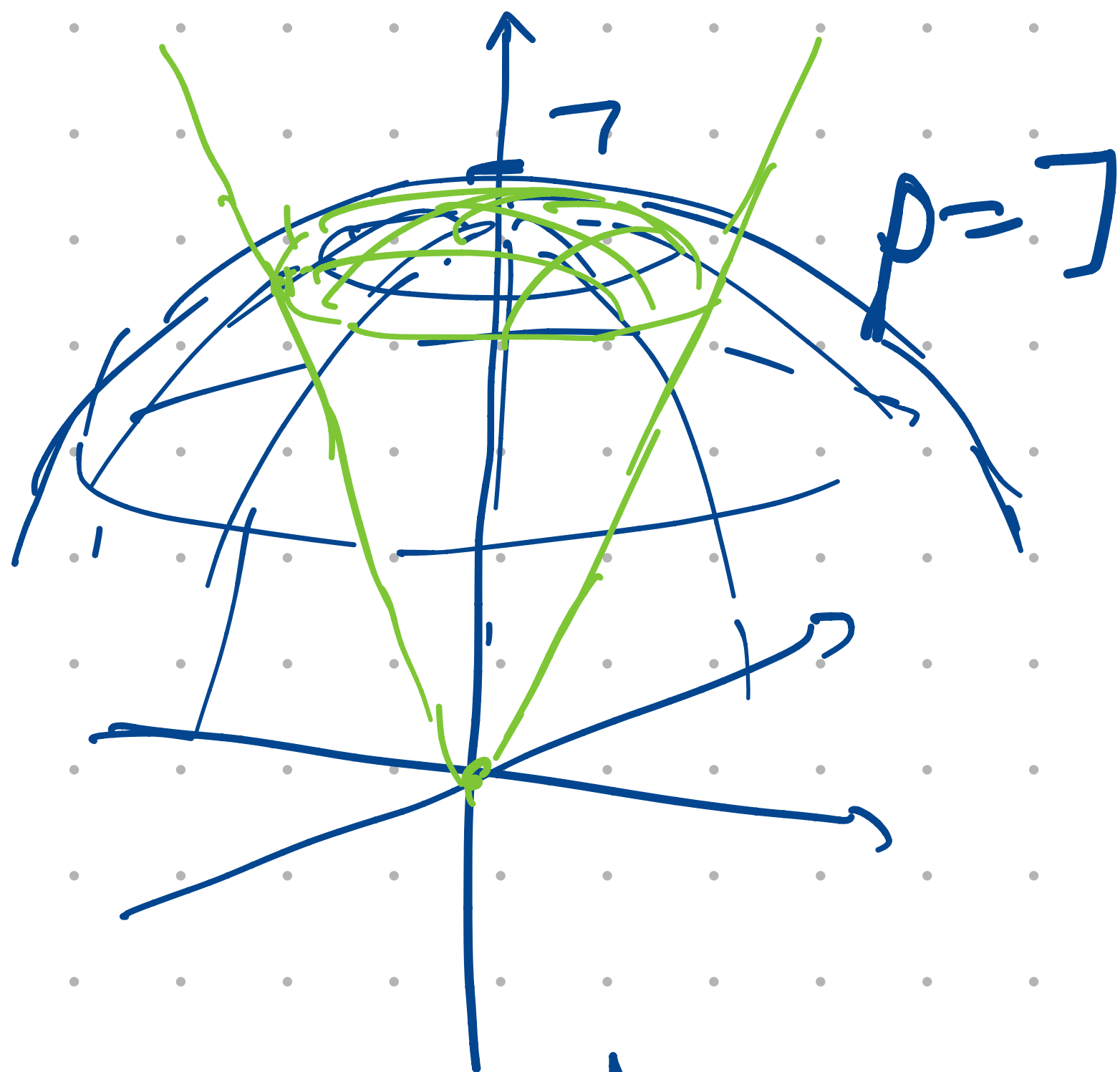
$$z \leq 7$$

So the region is a cone with a flat top





flat top



round top

#3) Standing assumptions for cylind: ($r \geq 0, 0 \leq \theta \leq 2\pi$)

"above $z = x^2 + y^2$ " means $z \geq x^2 + y^2$

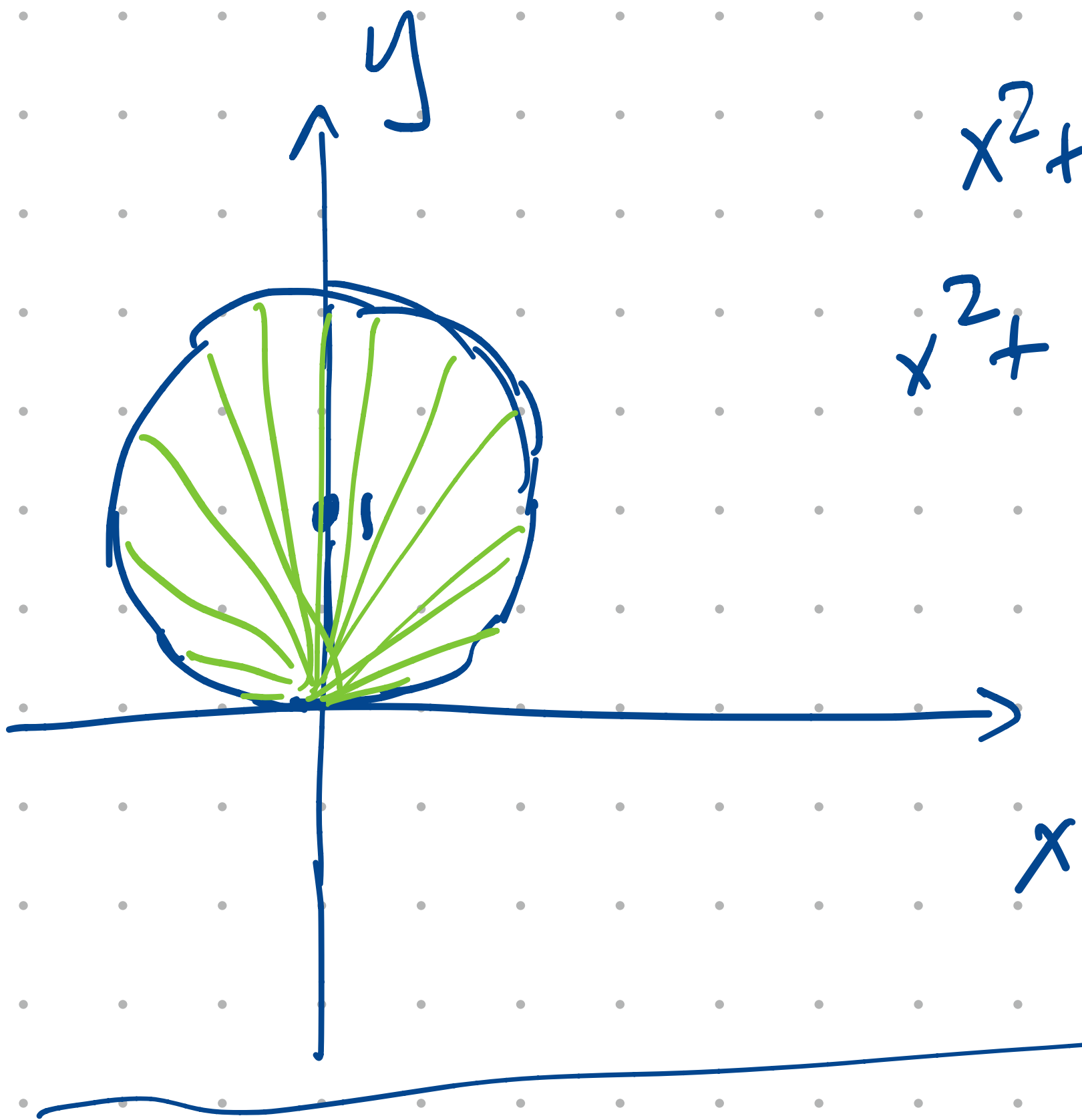
"below $z = 2y$ " means $z \leq 2y$

$$r^2 = x^2 + y^2 \leq z \leq 2y = 2r \sin \theta$$

z-bounds.

For r: $r^2 \leq 2r \sin \theta, \quad r \geq 0$

$$r \leq 2 \sin \theta$$



$$x^2 + y^2 = 2y$$

$$x^2 + (y-1)^2 = 1.$$

$$0 \leq r \leq 2 \sin \theta$$

For θ :

$$0 \leq 2 \sin \theta$$

$$0 \leq \theta \leq 2\pi$$

Get

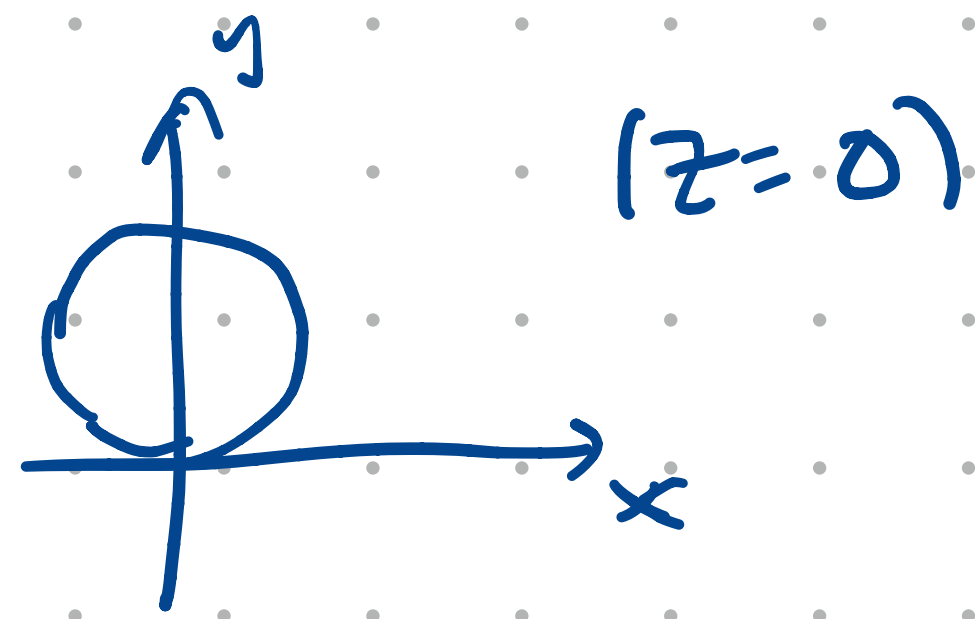
$$0 \leq \theta \leq \pi.$$

$$\int_0^{\pi} \int_0^{2 \sin \theta} \int_{r^2}^{2r \sin \theta} |r| \, dz \, dr \, d\theta$$

$$= \int_0^{\pi} \int_0^{2 \sin \theta} (2r \sin \theta - r^2) r \, dr \, d\theta = \dots$$

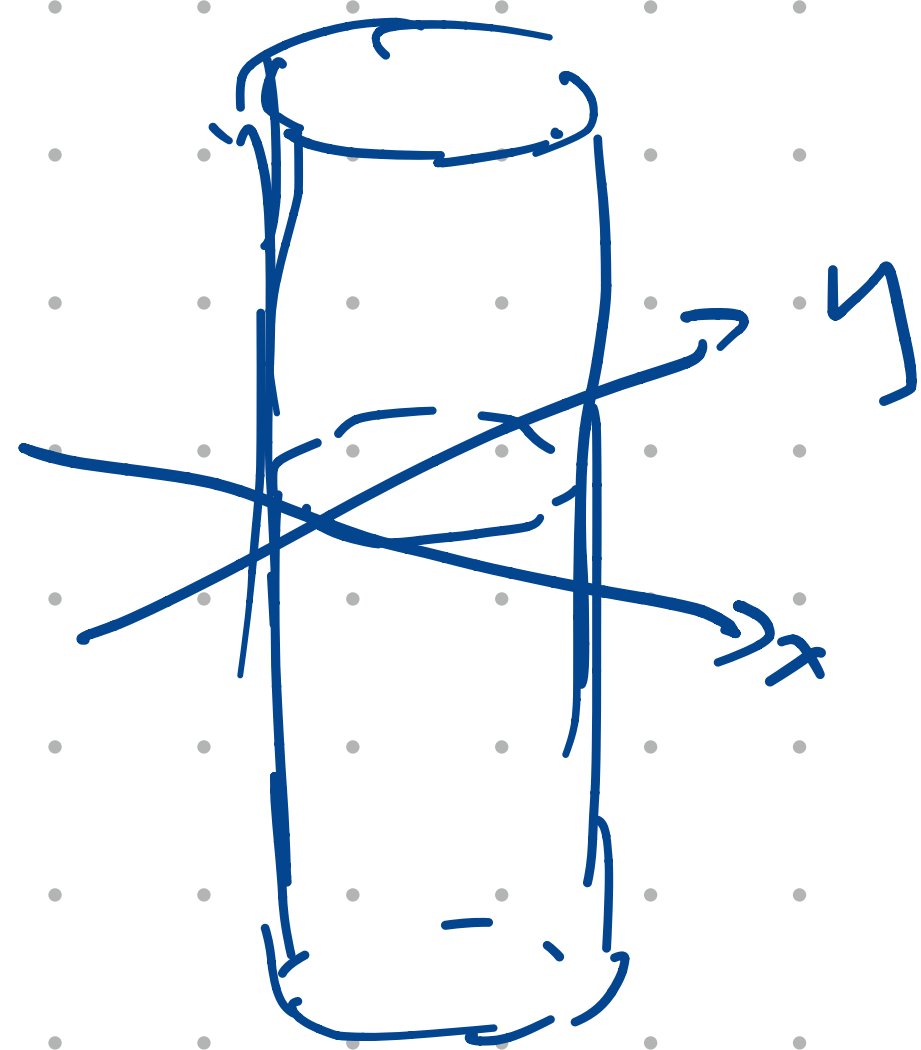
1) In polar (r, θ)

$$r = \sin \theta$$

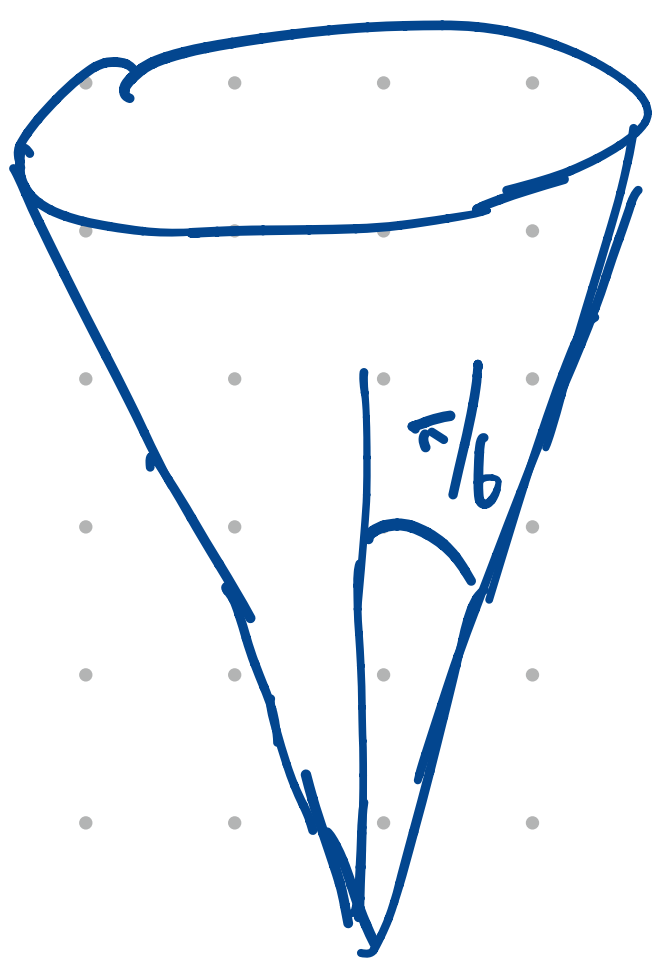


In cylindrical (r, θ, z)

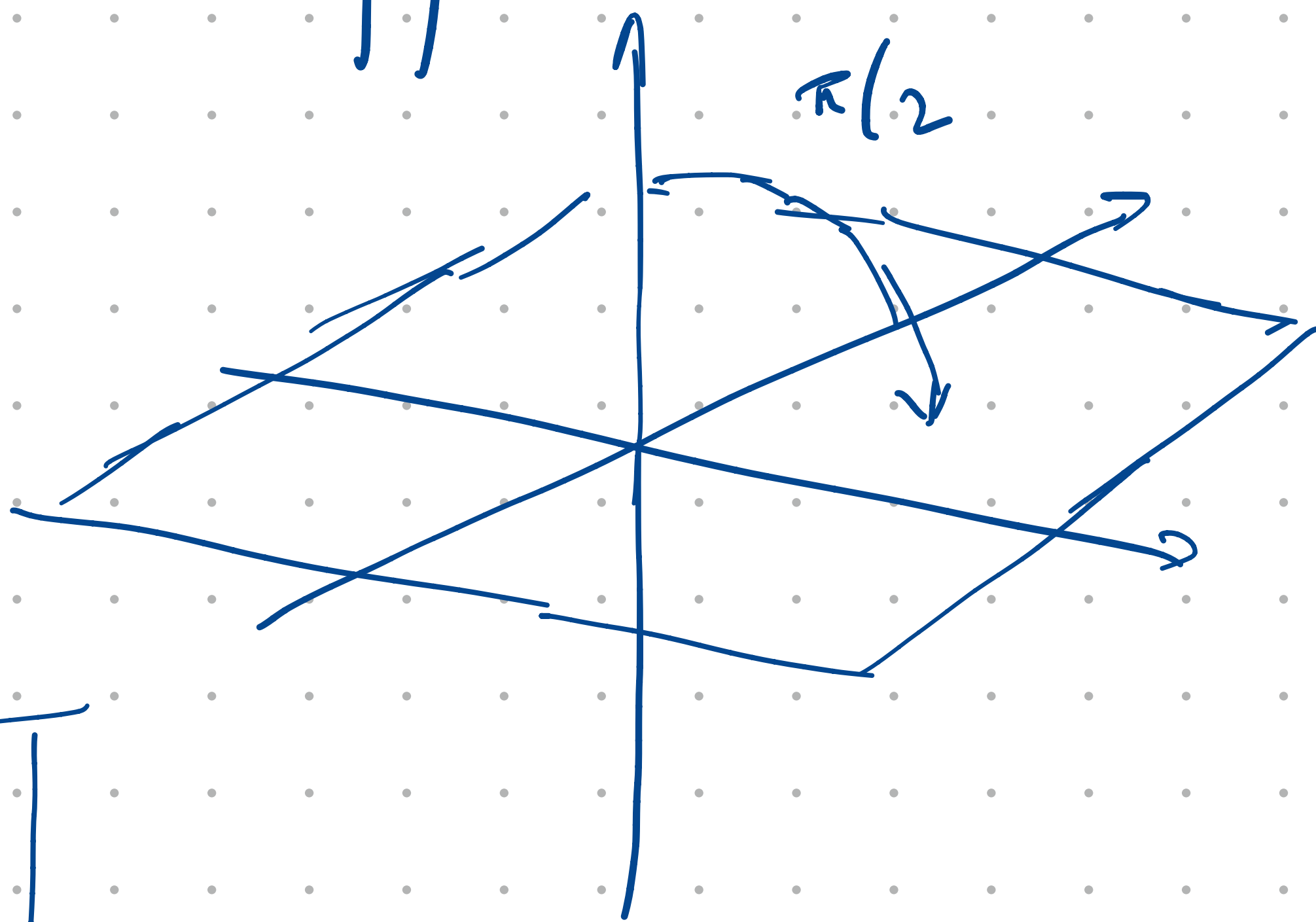
$$r = \sin \theta$$



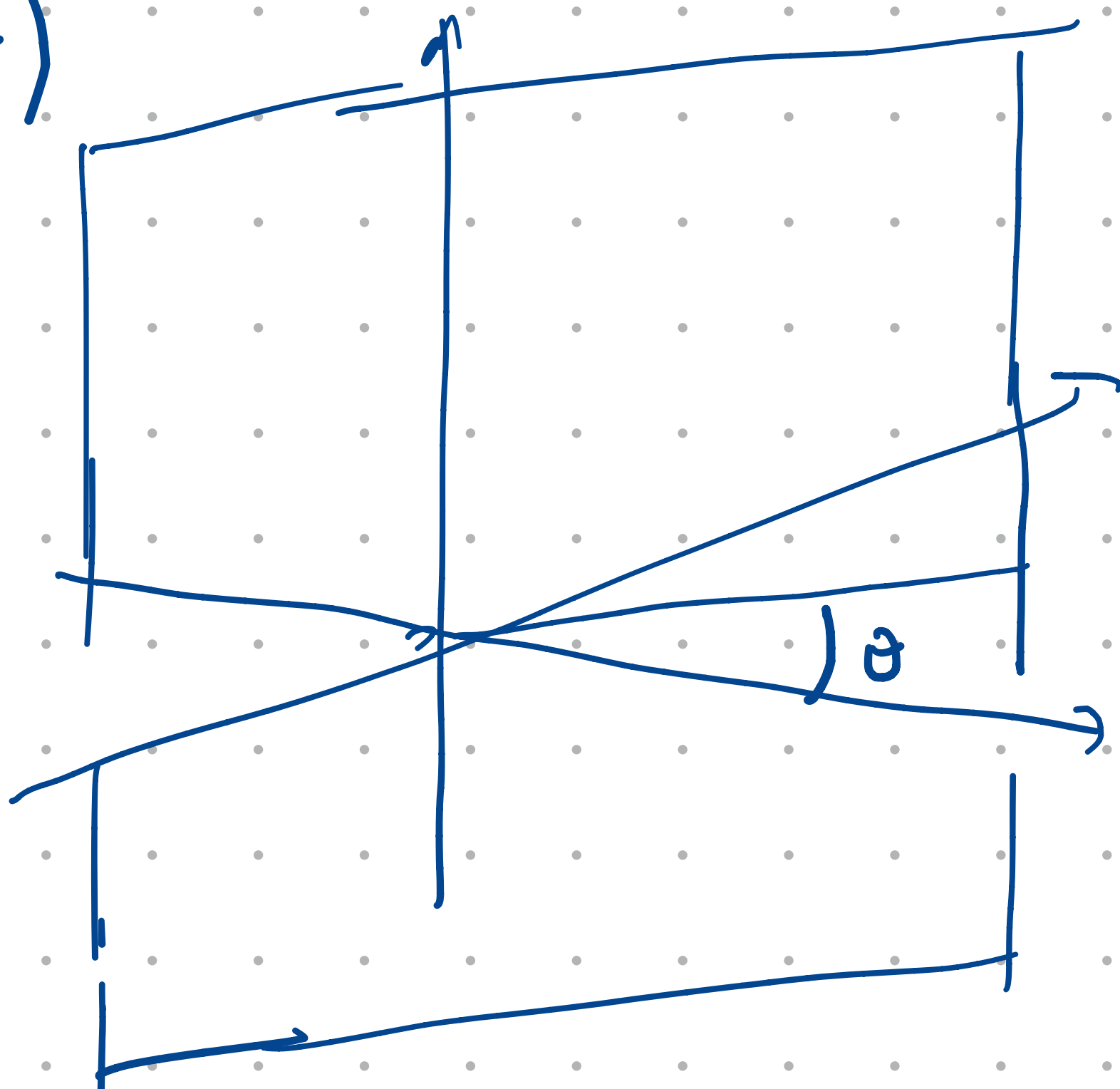
2) It's a cone (double cone)



3) $\phi = \pi/2$ is just the xy -plane!



4)



plane

$$5) \rho = \cos \phi$$

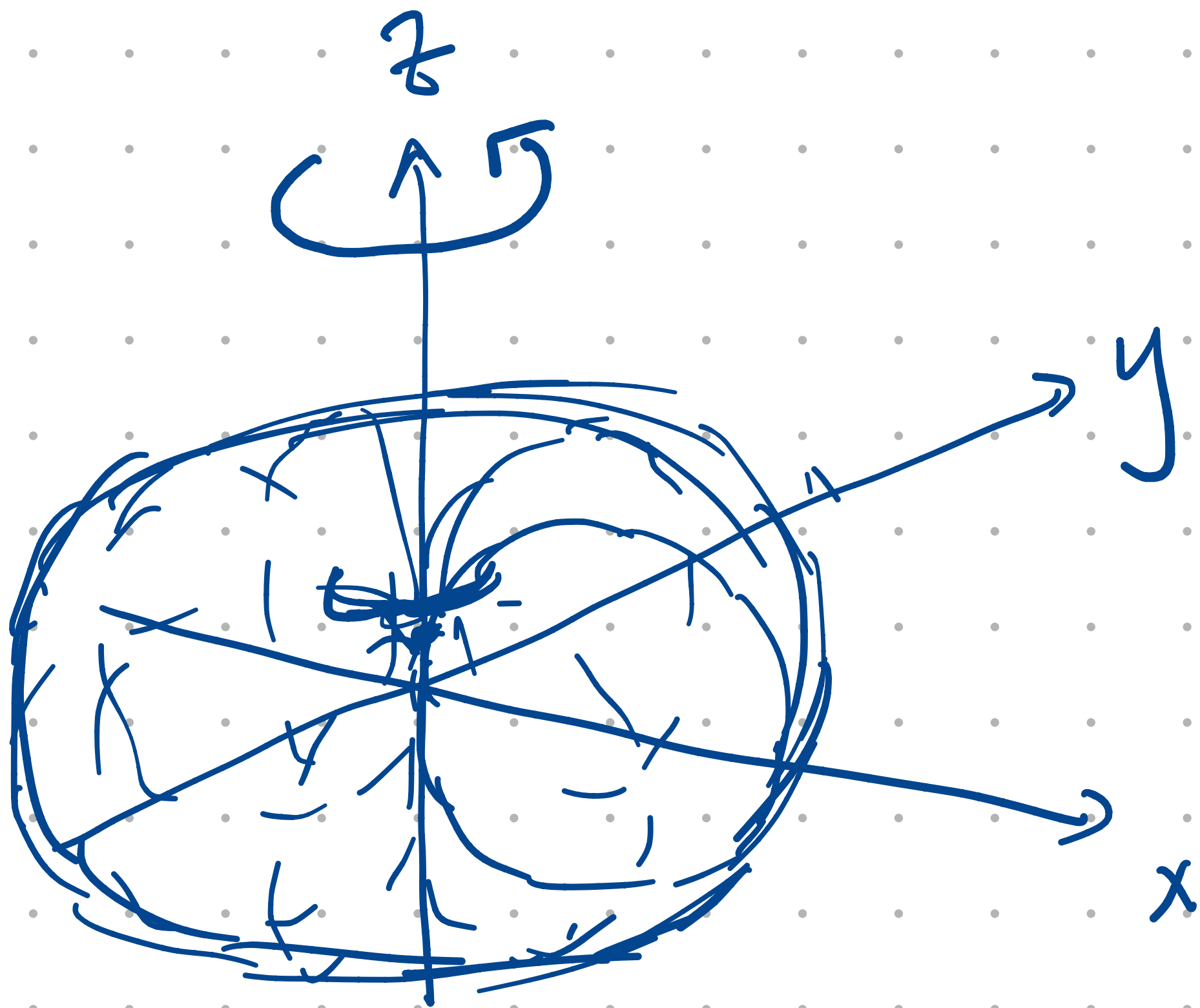
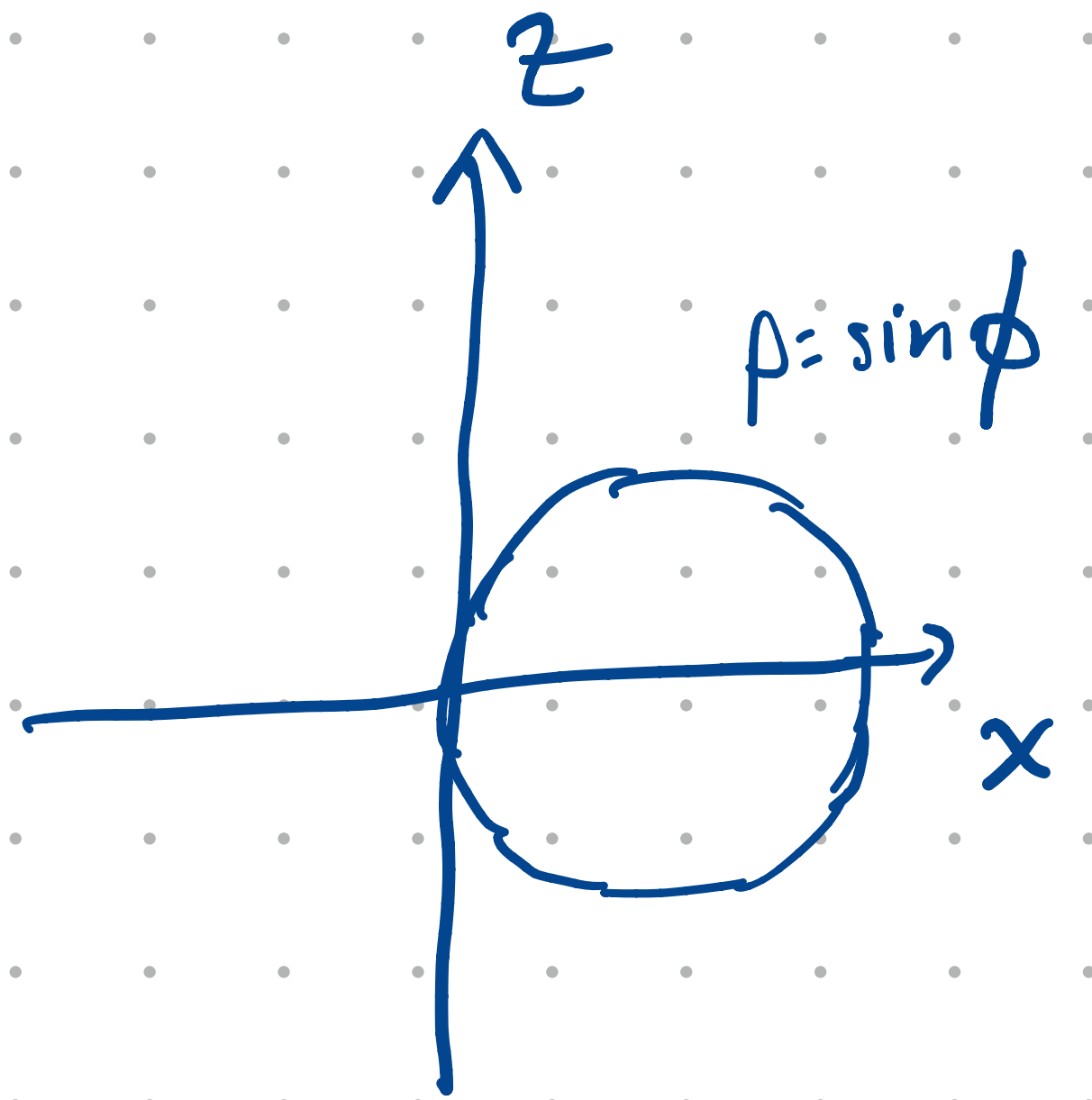
$$\rho^2 = \rho \cos \phi$$

$$x^2 + y^2 + z^2 = z$$

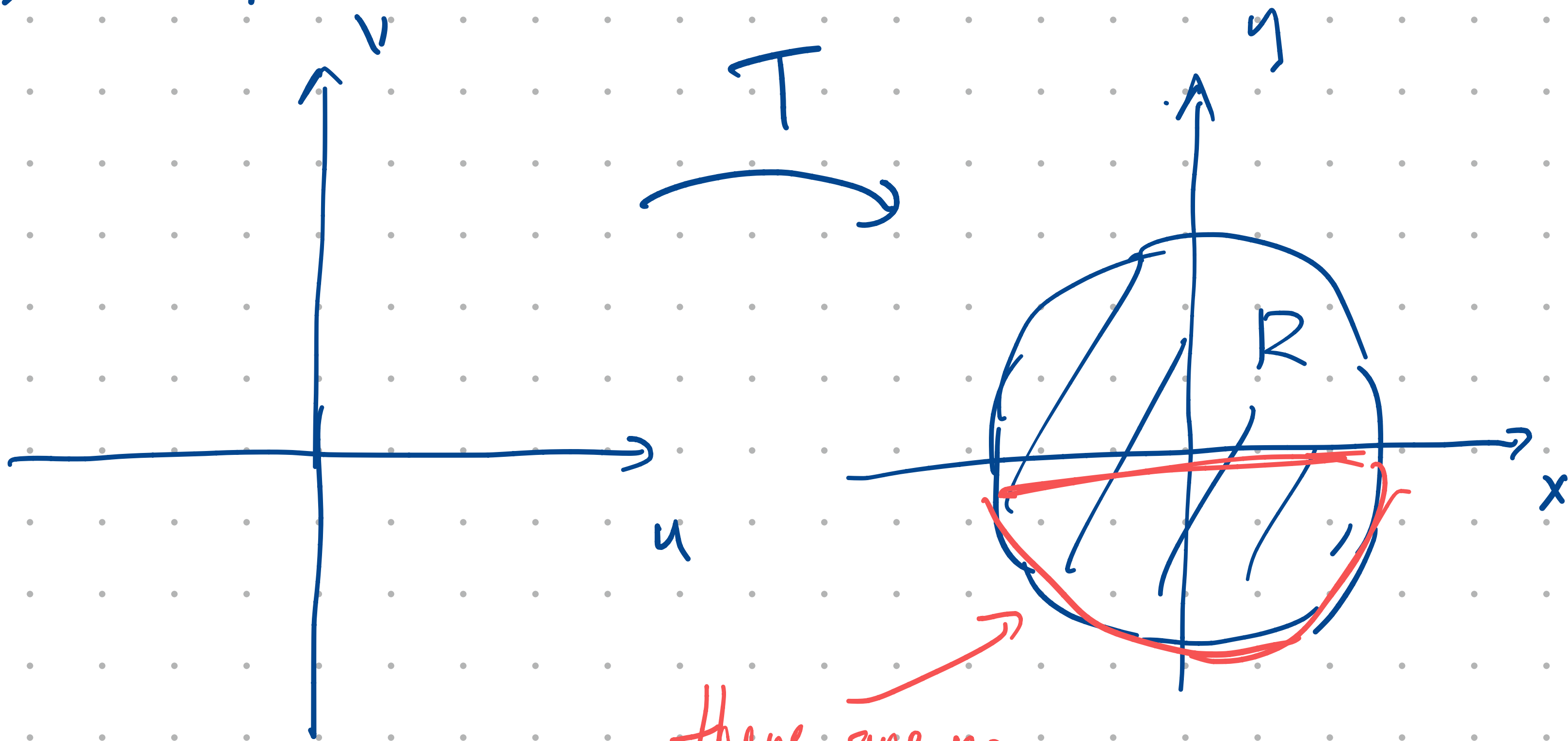
$$x^2 + y^2 + \left(z - \frac{1}{2}\right)^2 = \frac{1}{4} \quad \text{a sphere.}$$

$$6) \rho = \sin \phi.$$

For $\theta = 0$:



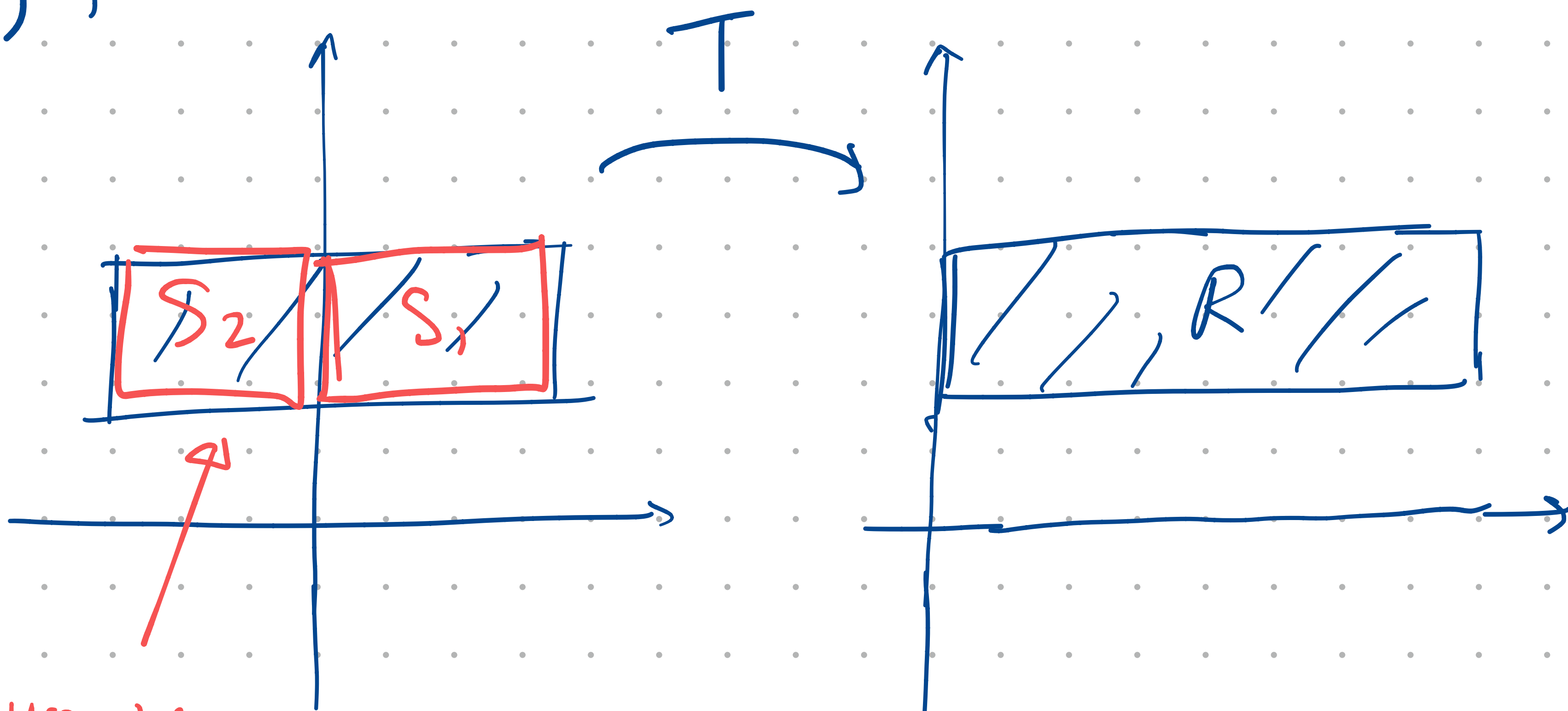
7) not fixable.



there are no solutions in the uv -plane for these!

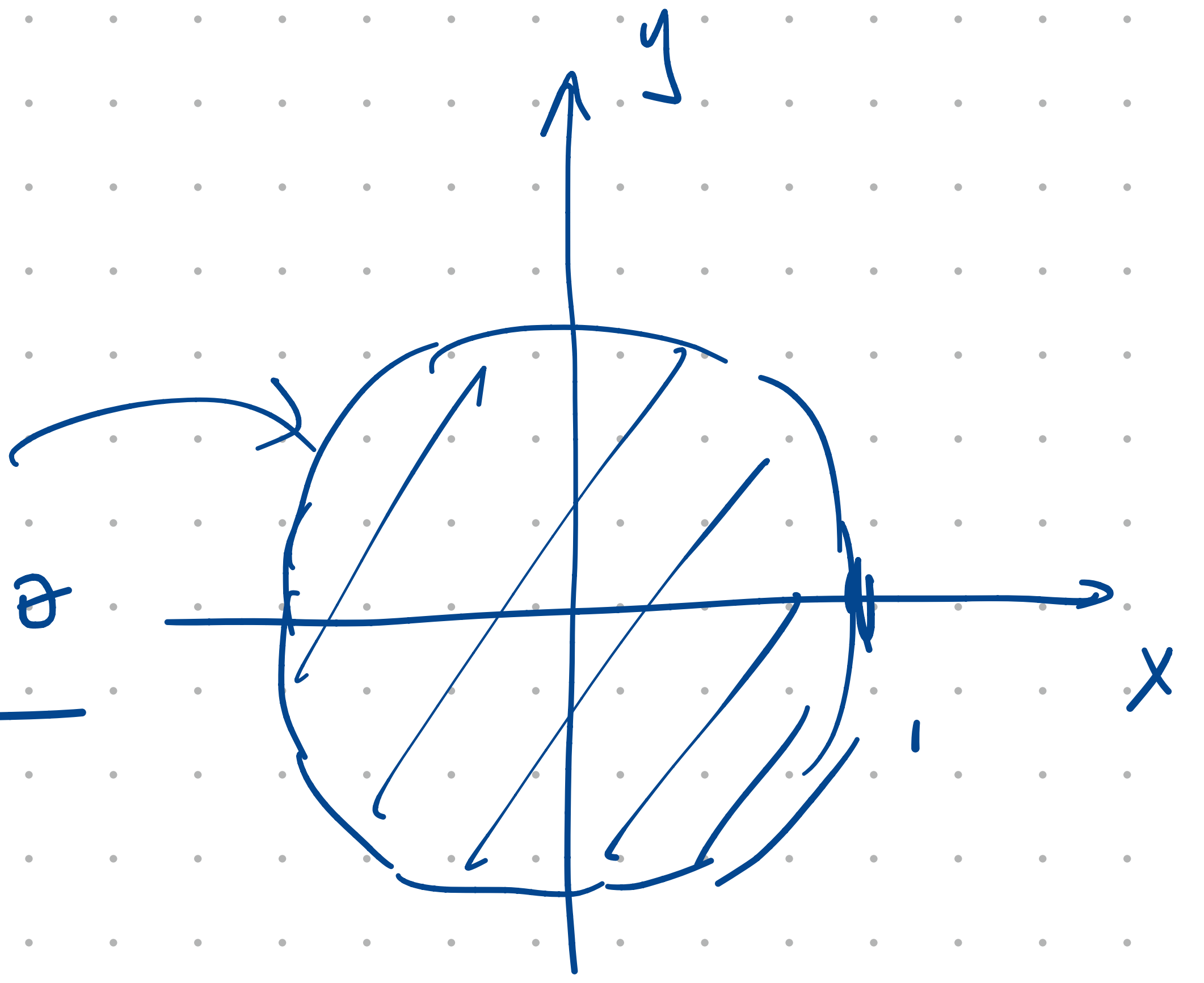
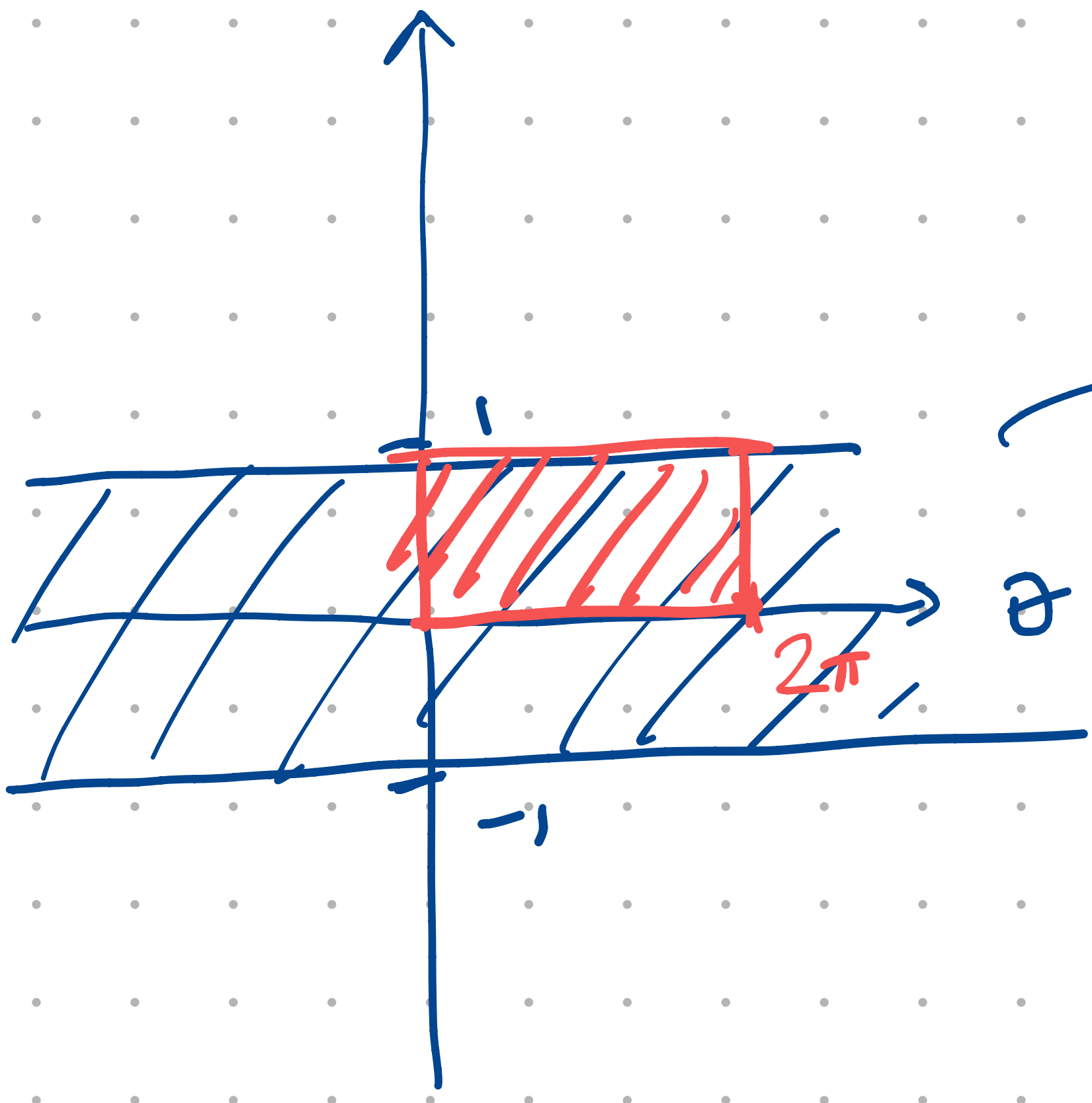
When changing vars, need exactly one solution in S for (u,v) , for every (x,y) in R

8) fixable.



Use one of these two regions, but not both!!

Another example: polar



(both rectangles are fine for changing var)

